Invited Lecture

Openness of Problem Solving in the 21st Century: Mathematical or Social?

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ABSTRACT Mathematics is one of the oldest disciplines in the world. Bishop (1991) expressed its value regarding human relationships and social institutions as Openness — that is, mathematical constructs such as propositions and ideas are open to human deliberation. Even before such systematization, many problems were solved and simultaneously created since earliest civilizations. This effort became the foundation for further endeavors.

What is the "Problem" in problem-solving? It has various types. Especially, the open-ended approach (Shimada, 1977) has been developed in Japan as a method to evaluate and develop mathematical thinking. Furthermore, problem posing can be an extension of problem solving. While posing various problems we may notice the patterns among those problem variations. In this sense, problem posing itself can be a problem. What is "Solving" in the problem-solving? It is dependent on the type and characteristic of problem. For example, the open-ended problems provide more than one solution. Socially open-ended problems provide solutions together with values. Problem posing requires developing problems and such development itself can be a solution. Therefore, importantly, the meaning of solving a problem is extended beyond traditional problem solving.

This paper explores the idea of problem-solving in mathematics to appreciate the value of openness under the Open Science movement (OSF, 2021). Open science is a movement accommodating experts and non-experts to have access to the outputs of scientific research and can participate in the research activities. This is essential for future citizens and is related to the ethical dimension of mathematics education (Ernest, 2012).

Keywords: Openness; Problem solving; Sociality; Mathematicality; Meta-problem.

1. Mathematical Value "Openness" (Bishop, 1991)

1.1. Background

Mathematics is one of the oldest disciplines in the world. Bishop (1991) described a set of values for mathematics and one of them is openness, which is related to human relationships and social institutions. It means that mathematical constructs such as propositions and ideas are open for human deliberation and they can be discussed

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among themselves. Even before the invention of such mathematical constructs, many problems were solved and created since the earliest civilizations. These efforts became the foundations for future endeavors.

During the Greek period, they considered paradoxes. “Zenon paradox regarding the infinity numbers of points on a line, threatened the certainty of mathematical science (Wilder, 1980, p.11).” This mathematical problem solving includes both “mathematicality” and “sociality”. Mathematicality refers to things related to content and method of mathematics. They include mathematical concepts such as algebra, calculus, and numbers, and mathematical process and method such as algorithm, proof, and calculation. Sociality means things related to context and method of society. They include social problem contexts such as purchasing at the market, surveying fields, and sharing food, and method such as discussion, debate, and voting. We analyze whether mathematical problem-solving involves sociality, in addition to its mathematicality.

Many mathematical problems started with the needs of society. For example, the mathematics in ancient civilizations such as Egypt and Babylonia was connected to weather forecast, cultivation, surveying and so on (Wilder, 1980; Cajori, 2015). Therefore, many problems do contain sociality. “Mathematics is what human beings create, and the form of mathematics, which human beings create is just a function of cultural need at that time, just like any other adaptation systems (Wilder, 1980, p.5).”

In the process of problem-solving, their interests shift from a specific problem to the general solution to solve problems with similar nature. Then how to get the general solution became the object for further consideration. Mathematics as scientific efforts began to use it to explore the nature of such objects. The most important thing in this (genesis of scientific concept) is that we can imagine any bigger numbers than those ever known in this physical world and proceed with studying on nature of such numbers once they are created (Wilder, 1980, p.9).

1.2. Contents of problem-solving

As we have seen, many mathematical problems have their root in society. From this perspective, problems in problem-solving contain sociality as a starting context. Eventually, some problems are even theoretically considered as they become an object of thinking through symbolization and formalization. It is surprising yet very natural that societies developed various numerals and later unified them through interaction. Following these symbols and number concepts, millions of similar contexts had been practiced and abstracted.

Besides the context in the beginning, sociality can also appear in the end — that is, application of mathematics into solution of social problem. In this highly technological society, mathematics is inseparably connected to science and technology; therefore, most social phenomena can be described through mathematical models. Mathematical models are an object of thinking, a starting point of consideration for the next stage. Such problems provide an opportunity for integrating mathematicality and sociality.

Therefore, the mathematicality and sociality of mathematical problem-solving can be discussed as starting contexts and application as its end. They are a part of content of problem-solving.

1.3. Method of problem-solving

Generally, solving a problem is goal of the problem-solving. However, if we further consider getting a general solution, we need to explore the process of problem-solving or how it was solved. "Thales (BC640–546) is an outstanding philosopher because he reformulated what many people accepted as truth into a theorem and further provided a proof for it (Cajori, 2015, p.76)."

Difficult problems such as Geometric problems of antiquity made us think of a general solution and system of logics such as Euclidean Geometry developed. Whether a certain statement would hold true or not for all cases is beyond the necessity of everyday life, because the people simply want to solve a specific problem in everyday life. However, philosophers at that time, called Sophists, discussed the need for such complete accuracy. "Athenians valued liberty and fairness in their life and Pythagorean custom of secrecy disappeared ... They wanted to prove themselves to be excellent through public debate regarding philosophy and science (Cajori, 2015, p.83)."

Here, a solution (method) is an object of consideration. From this perspective, method is called method knowledge. This age-old knowledge is being used even today. It is valuable social infrastructure for problem-solving, although we no longer remember the original context of such problem-solving.

1.4. Openness, mathematicality, and sociality

In the above mathematical problem-solving, not just one solution is sought, rather a general solution or how to solve problems is considered. In the latter, the developed general solution creates method knowledge such as "algorithms", "proof" and so on.

When general solutions and theorems were made, they might not be directly related to the necessity of the daily life. Later they might become social infrastructure and support scientific development. For example, quadratic equations were meant to calculate simply relation of areas, but later the general solution for it was developed due to theoretical necessity. Such a general solution further has become foundation for the theory of polynomial equation and description of ballistic path. In this sense, they later became a social necessity and were involved in society's daily life.

In the modern times, the school education has been systematized and the subjects have been established. The subject, social studies, was established much later to learn about the society. Distinction of mathematicality from sociality may strengthen the subject boundary. However, such distinction may result in refraining mathematics from perceiving society through a mathematical lens (mathematical literacy).

As noted earlier, the value openness represents human relationship and social institution. It is important to consider how the relation between mathematicality and sociality is established through openness as mathematical culture was practiced in the old days. “[T]he educational imperative is clearly there to demonstrate, and critically evaluate, the value of openness as represented by Mathematical knowledge (Bishop, 1991, p.77).”

2. Historical Development of Problem-Solving: the Case of Japan

Since this paper focuses on problem-solving in mathematics education, and sociality and mathematicality, and sociality may vary from one society to another, we pay attention to a specific society, Japan. We briefly review the history of mathematics education in Japan. Reference materials are Baba et al. (2012) and Ueda et al. (2014).

2.1. *Problem-solving (empiricism)*

After World War II, education reform was implemented. In senior secondary mathematics education, the “central idea” was proposed to cut across different fields within mathematics and later developed into mathematical thinking. On the other hand, in primary mathematics education, the life unit learning based on John Dewey’s philosophy used life events for the context of problem-solving.

For primary and junior secondary education during this period, the problem was presented based on a life event. For example, multiplication of fraction is introduced based on an episode of rice planting. At that time, Japan was predominantly an agricultural country and rice-planting was very prevalent across the country. The description of the context is very long and rich. Thus, during this period, the focus of problem-solving is placed on sociality. It seemed natural for Japan to have such focus due to a scarcity of natural resources. However, there was a criticism regarding as lowering the achievement of students (Kubo, 1951) and this approach was suspended suddenly.

2.2. *Open-ended approach*

In 1958, the national curriculum, the Course of Study, was published for the first time. The term “mathematical thinking” first appeared as an objective at the primary school level (Baba et al., 2012). During this period, the focus shifted more to mathematicality. Soon after, because of the influence of modernization of mathematics in the USA, focus on mathematicality was further strengthened. The issue at that time was how to evaluate this mathematical thinking.

The open-ended approach (Shimada, 1977) had been developed in 1970s as a method to evaluate and later grow mathematical thinking. This open-ended approach uses an open-ended problem (Fig. 1), which has more than one solution. Shimada (1977) provides a theoretical background and a compilation of examples of such problems.

Open ended approach is to set an open-ended problem as the task, to utilize proactively its various solutions, to combine in various ways the previous knowledge, skills and ideas …, to give students an experience in which they have found new things (Shimada, 1977, pp. 9–10).

How many marbles are there?
Find out the solution in various ways.

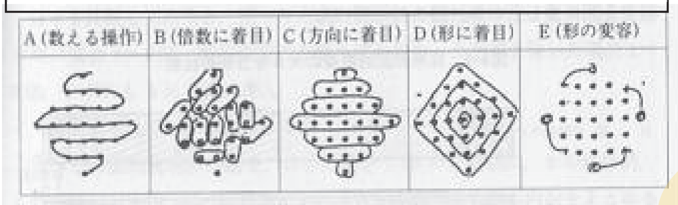

Fig. 1. Example of open-ended problems (Shimada, 1977)

In 1980s, the systematization of an open-ended approach, which was called Learning through Problem solving (Kadai-gakushu), was proposed. This clearly separate “finding a solution to the problem” and “learning something through solving the problem”. In other words, it is important whether finding a solution is the purpose or method of problem-solving. If it is the method, the purpose should be set appropriately; thus, mathematical thinking reappeared here again.

Treatment of more than one solution was an important topic. Koto (1990) summarized the treatment of various solutions into four types. Not only Kadai-gakushu and the treatment but also other approaches became extensively practiced and studied. Problem-solving has become an integral part of mathematics lesson in Japan. Such efforts have developed a unique characteristic of lesson in Japan as a “structured problem-solving lesson” (Stigler and Hiebert, 1999).

2.3. ***Problem posing***

Further, problem posing can be an extension of the problem-solving. While varying problems systematically, we may be able to pose as many problems as we want, and realize the patterns among such variations. In this sense, asking for problem posing can be a problem in itself and posed problems are a solution to it.

Walter and Brown (1983) proposed the approach called “What if not” for problem posing. A part of the original problem is changed by asking “what if not”. This approach consists of steps such as a starting point, listing attributes, “what-if-not”-ing, question asking or problem posing and analyzing the problem. Takeuchi and Sawada (1984) proposed another approach called “From a problem to a problem”. Takeuchi (1976, pp. 11–12) employed the theory of scientific knowledge growth by Popper and approached this issue from the perspective of the nature of mathematical activity. This played an important role in shifting the research from the open-ended approach to “extensive treatment of problems.”

This assumes that “Existence of problem causes cognitive activity. Cognition develops knowledge. Progress of cognition and knowledge is brought by self-proliferation. In other words, it is a chain reaction of from a problem to a problem (Takeuchi and Sawada, 1984, p.15).” In practice, they segmented the original problem into parts and alter each part with other phrases. For example, the original problem is “How many diagonals are in the regular octagon?” The part “diagonal” can be replaced with “side”, “angle”, and so on.

Nohda (1983) summarized the openness in problem-solving into three types. The first one is “End products are open.” This type has multiple correct answers. Shimada (1977) and his colleagues have been developed this type of problems. The second one is “Process is open.” This has multiple ways of solving the original problem. It is needless to say that all mathematical problems are inherently open in this sense. The last one is “Ways to develop are open.” After students solved the problem, they can develop new problems by changing the conditions or attributions of the original problem (Takeuchi and Sawada, 1984).

2.4. *Mathematical literacy*

Mathematical literacy is defined as an individual’s capacity to reason mathematically and to formulate, employ and interpret mathematics to solve problems in a variety of real-world contexts. It includes concepts, procedures, facts and tools to describe, explain and predict phenomena. It helps individuals know the role that mathematics plays in the world and make the well-founded judgments and decisions needed by constructive, engaged and reflective 21st century citizens (OECD, 2012, p.100).

Here, the students are expected to make an interpretation using mathematical modelling. Sociality appears explicitly once again in the history.

Lesh and Zawojewski (2007, pp.783–784) describes “the problem solver will engage in ‘mathematical thinking’ as they produce, refine, or adapt complex artifacts or conceptual tools that are for some purpose and by some client.” This kind of

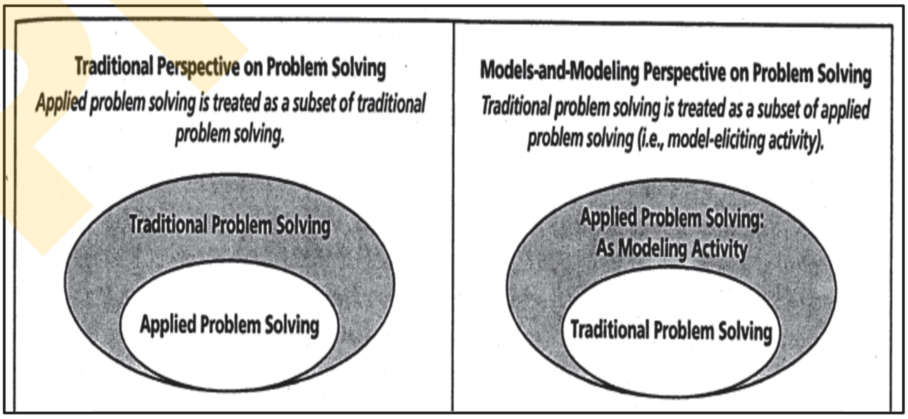

Fig. 2. Modelling (Lesh and Zawojewski, 2007, p.783)

problem-solving is called “model eliciting activities (MEA)”, includes traditional problem-solving itself, and makes mathematical sense of problem solution (Fig. 2).

2.5. *Shifting between mathematicality and sociality.*

Going through the history of problem solving in mathematics education and, we can see the shift of focus between mathematicality and sociality from time to time. Simultaneously, due to theoretical development of mathematics education research, both sociality and mathematicality are treated more sophisticatedly than before. Especially, MEA involves both in an integrated way. There are two important points to consider.

First, integration of both mathematicality and sociality. As we have seen, the idea of MEA plays such a role. Besides, there are many variations. Our research group (e.g. Baba, 2007; Shimada and Baba, 2015; Hattori et al., 2021) also has tried to extend the open-ended approach by paying attention to the values and the sociality. We call them social open-ended problems, and the students provide solutions with various values. This is further discussed in the chapter 4.

Second, the societies in 1950s and in 2020s are considerably different even in some countries. This is important because we deal with sociality, and it varies from one society to another as well as over time. Thus, it is necessary to consider time and space.

3. What is a Problem and Problem-Solving in Today’s Society?

3.1. *Traditional problem-solving and new problem-solving*

Through reviewing the historical development of mathematics education in Japan, we realize that problem solving has occupied a central position throughout its history and has changed its approach and focus in and of itself. Therefore, although we use the term “problem solving” extensively, it may not mean the same thing. Thus, it is crucial to be conscious about the meaning of the word.

In the traditional problem-solving (in the left of the Fig. 2), solving the problem is the purpose, while in a new way of problem-solving (in the right of the Fig. 2), “mathematical ideas and problem-solving capabilities co-develop during the problem-solving process” (Lesh and Zawojewski, 2007, p.783).

Here, we would like to consider the etymological origin of the word “problem”. Since “pro” means “forward” and “ballein” means to “throw”, the word means “anything thrown forward”. It does not necessarily have a negative connotation. Especially the solution in the new way of problem-solving may open a new way of interpreting the situation. If it would lead to a later development, identification of the problem is a first step of development.

The current society is called a highly technological society and/or highly information-oriented society. In this society, various advanced technologies and ICT, which connect such technologies, occupy a central position. Unimaginable things may be becoming reality with innovation of technology. However, such technology may generate problems at the same time. For example, we manifest some cases in which an incurable disease become curable due to an advancement in medical science. Certainly, simple extension of our life does not necessarily mean good. There are people who do not appreciate prolonged life if that entails their staying in bed longer. This is related to the life which is very fundamental for us all. Certainly, life at individual level may be considered and decided by the individual person.

There are many more problems which we face due to technological advancement. It includes leakage of private information, control of freedom of expression in SNS (Social Networking Services), and so on. They are not related to life but still significant in our life. They are called trans-scientific problem (Kobayashi, 2007). Most of us are non-scientists or non-experts but should find out some solution. We should decide among some alternatives by negotiating different views and need to establish a new system of deliberation among citizens.

Thus, in the new problem solving — MEA, "the solution (artifact, tool) problem solves create embodies the mathematical process they constructed for the situation (Lesh and Zawojewski, 2007, p.784)." We can realize and interpret emergence of various model eliciting activities such as socio-critical modelling (Barbosa, 2006; Dede, Akcakin, and Kaya, 2020), and ethno-modelling (Rosa and Orey, 2010) due to the complexity of this highly technological society. They form a bridge between mathematicality and sociality.

3.2. *What is the range of problems in problem-solving?*

It is impossible to understand all advanced technologies. We may be overwhelmed with intricate details of them. Therefore, it is important for us to grasp an essential problem surrounding them. This may be an ability of problem-posing in the broader sense.

What does "solving" mean in the problem-solving? As there are different types and characteristics of problems, the meaning of "solving" also depends on their types. If the problem has only one correct answer, then solving problem means to get exactly that answer. If the problem is open-ended, solving the problem may mean to get all answers which satisfy the condition. If the problem is to pose problems by changing the original problem, solving the problem may not necessarily mean to get a problem and problems as an answer but to understand the characteristics of the original problem structure and to pose as many problems as possible through creativity. An important point here is that the meaning of solving a problem is extended beyond traditional problem-solving.

Then, an important question is "how do we deal with the extension of problem-solving?" Here openness may be keywords. We may have to go back to the trans-scientific problems.

[social openness] Considering today's society as highly technologically advanced and highly information oriented, the idea of "trans-scientific" problem (Weinberg, 1972; Kobayashi, 2007) indicates a crucial relation between science and society. This is the problem which “can be asked of science and yet which cannot be answered by science” (Weinberg, 1972, p.209).

[evaluation openness] The trans-scientific problem requires not finding only one logically correct answer but evaluation of alternative solutions.

It is important to think subjectively and engage with society through viewing the society mathematically and thinking mathematically in solving social problems. Therefore, it is impossible to teach all necessary mathematics concepts and skills in advance. Of course, it is a minimum requirement to master basic mathematical ideas and acquire skills of applying them into a problem.

The model eliciting activities are a new mode of problem-solving. Solving a problem in the model eliciting activities means to deal with problems, to develop mathematical models based on the given conditions and to evaluate the alternatives. It can include trans-scientific problems. Here to-and-fro motions are evident for bridging between mathematicality and sociality. Then how far should we deal with, if the range of problems to be dealt with is ever-expanding?

To grasp the situation holistically, content-based mathematical thinking (e.g., proportional reasoning, functional literacy, linearity, exponential and logarithmic thinking) is necessary. To see the situation and make a decision, we need to acquire such a mathematical way of viewing supported by mathematical thinking. More importantly, deductive and logical thinking is another important asset of mathematics, while natural science and statistics are basically inductive. Thus, it is important for students to understand deductive reasoning which is invented by human beings and its difference from inductive reasoning.

Henceforth, the ability of dealing with big data is necessary. However, if the people are not cautious enough, they simply believe in the result which the computer software gives. Or, once the people are given a percentage such as 95% and 99.9%, they may believe it is high and it is perfect. We cannot easily say “it is absolutely ...” and may be confused by the expression “it is significant with 95%.” We may call this as a logical contextual thinking because not just logical thinking but also judgement based on the context are important.

3.3. *How to deal with the problems*

The model eliciting activities are to solve a problem, using logical contextual thinking. In this sense, we are able to see varieties of model eliciting activities as stated previously. Here are two to-and-fro motions:

(1) A to-and-fro motion between mathematicality and sociality

Fig. 3 below shows the statistical problem-solving and deals with the real world through problem and data. Fig. 4 shows a cycle of mathematical problem-solving and deals with the real world more directly. They develop, analyze and discuss mathematical model as a dynamic activity. This implies an important point when children learn to acquire mathematical knowledge and skills and to apply to solve the problems. It is crucial to create cycle between knowing why mathematical thinking functions as it is and creating how well such thinking is applied to solve the problem.

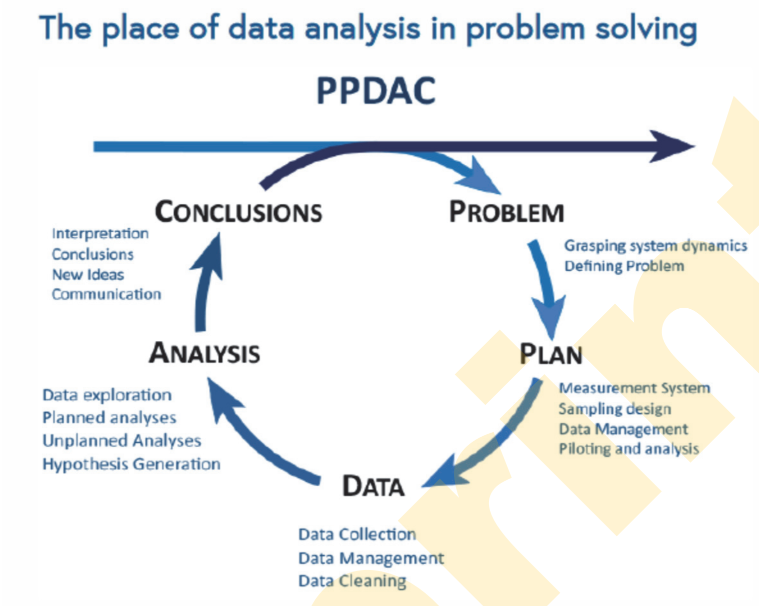

Fig. 3. Statistical problem solving (Wild and Pfunkuch, 1999)

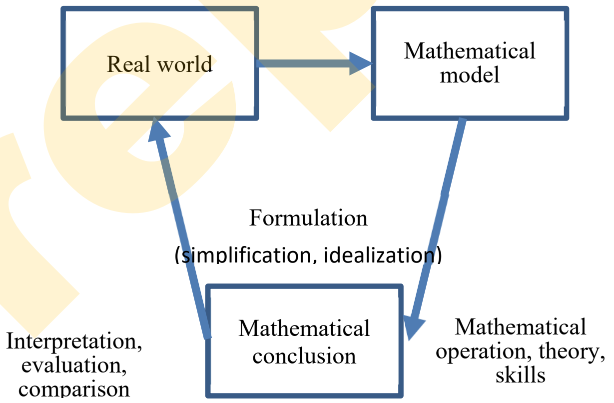

Fig. 4. Mathematical problem solving (Miwa, 1983)

After graduation from formal education, there is no longer a framework of subjects for thinking through. That is why the children need to master not only subject knowledge and method bounded by the subjects but also methods and attitudes to think and solve problems beyond subject borders.

(2) A to-and-fro motion between product and process knowledge

In both, we see the real world through mathematical lens (process) and manipulate mathematical model (product). Relation between process and product has been emphasized such as “procept” (Gray and Tall, 1994) and “objectification of method” (Hirabayashi, 1978).

Katagiri (1988) classified mathematical thinking into those related to methods and contents. Furthermore, mathematical thinking plays more significant role not as distinct entities of methods and contents but as an integrated form of them.

Based on these two to-and-fro motions, what role does mathematics play in relation with society? Who is required with how much mathematics? What kinds of social problems do we deal with mathematically?

What is the problem in creating these motions and to facilitate proactive and deep learning? It does not automatically guarantee that to solve mathematical problems related to society will create such learning. Rather, it is our task to think how to ensure such learning intentionally and systematically as an extension of problem-solving.

3.4. *Problem and meta-problem*

Therefore, it is important to ask what kind of problem can promote such learning. This is a kind of meta-problem that is “a problem about a problem” (Chalmers, 2018). Examples of meta-problems are “what kind of problems do we deal with?”, “Why do we deal with them?” and so on.

Here are two levels in relation with problem solving. One level is called an “object level of solving a problem.” This is usually the level of problem solving. Solving a given problem belongs to this level. Although the meaning of solving may vary depending upon the types of problems, at least problem and solution correspond each other. On the other hand, the other one is “meta-level of solving a problem.” (Fig. 5). Thinking about the meaning and reason of solving a problem belongs to this level. We expand the range of problems in relation with society and consider what problems and why. This thinking facilitates students to acquire not only problem-solving skills but also viewing the real world through problem-solving.

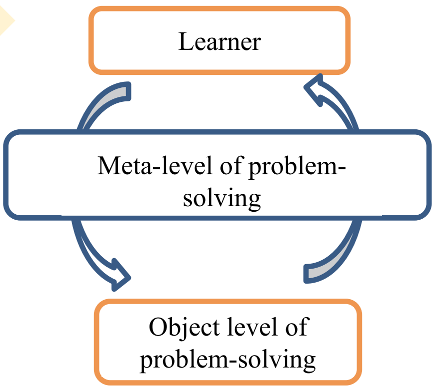

Fig. 5. Problem and meta-problem (Author created)

4. What will Be a Problem and Problem-Solving in Future?

4.1. *Openness in open science movement*

This chapter introduces the idea of Open Science movement (OSF, 2021) and explores its relation with problem-solving in mathematics. Open Science represents "a new approach to the scientific process based on cooperative work and new ways of diffusing knowledge by using digital technologies (European Commission, 2015, p. 33)." Because the impact of science on the society is getting more serious, it is more important not only for experts but also for non-experts to participate in the research activities.

Here, participate does not mean the same thing for experts and non-experts. And openness means different aspects of science such as openness in methodology, source, data, access, peer-reviewing, and educational resource. One example is methodological openness. Citizen science is scientific research conducted, in whole or in part, by amateur scientists. It is sometimes described as "public participation in scientific research", participatory monitoring, and "participatory action research by improving the scientific communities" capacity, as well as increasing the public's understanding of science. Another example is open access journals are to ensure everyone's access to the research. Since the journal publication becomes huge industry, it becomes too commercial and sometimes impacts on the research ethics. As a result, the interests of society and citizens may be at risk.

On the other hand, openness in mathematics means "With rationalism as an ideology and progress as the goals, individuals are liberated to question, to create alternatives and to seek rational solutions to their life's problem (Bishop, 1991, p. 76)." These practices of mathematics contain open discussion and alternatives.

Here openness shows the social aspect of problem-solving. Social aspect is not only related to the problem content but also solution method and reason. "… they (Greeks) develop the skills of articulation and demonstration in Mathematics (Bishop 1991, p.75)." It is important to explain and discuss rationally. This concerns openness, "relationships between people, and within social institutions (Bishop, 1991, p. 75)," and an ethical dimension of mathematics education (Ernest, 2012; Atweh and Brady, 2009).

4.2. *A case study: hitting a target*

Here we take one case from our recent efforts. The "Hitting a target" given to Grade 4 students at school (Shimada and Baba, 2015) is a socially open-ended problem (Fig.6).

At a school cultural festival, your class offers a game of hitting a target with three balls. If the total score is 13points or more, you can choose three favorite gifts. If you score 10 to 12 points, you get two prizes, and if you score 3 to 9 points, you get only one prize. A first grader threw a ball three times and hit the target in the 5-point area, the 3-point area, and on the border between the 3-point and 1-point areas. How do you give the score to the student?

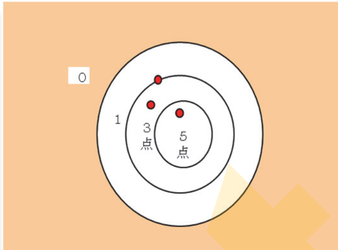

Fig. 6. Matoate problem (Shimada and Baba 2015)

They understand the problem and develop following mathematical models and values after the individual problem-solving activity. Some of them focuses on kindness because the player of the game is the first grader, and they tend to be kind to small children. Others focus on fairness, but such values did not appear explicitly at first. It became explicit only after being compared with kindness group (Tab. 1).

Tab. 1. Mathematical models and values (Shimada and Baba 2015)

Mathematical model	Expected value type	Explicit value (%)	Explicit value (%)
a. 5+3+3	Kindness to the first grader (Specific person)	92.9 (13/14)	94.4 (17/18)
b. 5+3+(3+1)		100.0 (1/1)	
c. 5+3+3+1+1		100.0 (1/1)	
d. 5+3+2		100.0 (2/2)	
e. 5+3+2	Fairness and equality to the whole class (all students)	0.0 (0/9)	0.0 (0/20)
f. 5+3+1		0.0 (0/10)	
g. 5+3+3		0.0 (0/1)	

After these mathematical models and values are presented to the whole class, the discussion started among the students. They ask questions to understand others' ideas and others explained their opinions. After discussion among the whole class, all students were asked to choose a model and a value again at the end. Some students have changed their opinions and while others maintain their opinions. Among those who changed their opinions, a few of them polished mathematical models.

This case is reflected from the perspectives of openness and open science. Regarding the value openness of mathematics, three points can be considered.

(1) Open-ended problem generally ensures multiple answers and solutions. This case stimulates students to have mathematical models based on their own values.

(2) The problem contains social context and promotes students to think more realistically. Thus, the solutions may contain mathematical models based on some values. This can be referred to as the social openness of the problem.

(3) The discussion is open to all students. They enjoyed mathematics and some of them even changed their opinions by agreeing with the others' explanations. This openness polished their models as well. It creates a culture of mathematics classroom.

Regarding open science, especially methodology, this case contains two meanings of openness.

(1) The first is to share and discuss their own mathematical models and values in a classroom. Through comparing and discussing, they realized the existence of different ideas and agree/disagree with the different ideas. This is a foundation for open discussion as a method.

(2) The second is to explore different ideas for oneself. Self-reflection enhances fluency, uniqueness, and originality to develop creative ideas. This self-exploration and self-reflection can facilitate students being aware of different ideas and appreciating the value of those ideas.

From these, individual exploration and mutual discussion are materialized in this classroom and the to-and-fro motion between sociality and mathematicality is ensured in the process. Especially, Matoate problem contains values such as kindness to those not proficient and impartiality. This is related to ethnical dimension of mathematics education (Ernest, 2012; Skovsmose, 2018) and is essential for future citizens. Thus, sociality appears both in method and content of problem solving.

4.3. *Concluding remarks*

Mathematical value and open science are connected to each other via "openness." At the base of mathematical problem-solving, there is a connection between mathematicality and sociality and thus openness. Here MEA is a new type of problem-solving using these. Relation between problem (object level) and meta-problem (meta-level) can offer a theoretical discussion about openness and problem-solving in mathematics education. This relation is related to the to-and-fro motion between mathematicality and sociality and another motion between content and process. Students appreciated this motion during the problem-solving although sometimes they feel it is beyond their mathematical problem-solving.

One solution to the meta-problem is the category of rulemaking which this Matoate problem belongs to and involves judgement and calculation. We may further ask if there are any other problems in this category and different categories from rule making. Problem and meta-problem are connected in this way.

Because society and time have sociality and mathematicality more intermingled, integration of both, rather than separation, should be carefully examined by avoiding their careless mixture. "… the educational imperative is clearly there to demonstrate, and critically evaluate, the value of openness as represented by Mathematical knowledge (Bishop, 1991, p. 77)."

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